



UNIVERSITY OF  
GOTHENBURG

# Synchronization problems with semaphores

Lecture 4 of TDA384/DIT391

Principles of Concurrent Programming

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*Based on course slides by Carlo A. Furia and Sandro Stucki*

# Today's menu

Dining philosophers

Producer-consumer

Barriers

Readers-writers

# A gallery of synchronization problems

In today's class, we go through several **classical synchronization problems** and solve them using threads and semaphores.

If you want to learn about many other synchronization problems and their solutions, check out "[The little book of semaphores](#)" by A. B. Downey available at <http://greenteapress.com/semaphores/>.

We will use **pseudo-code**, which simplifies the details of Java syntax and libraries but which can be turned into fully functioning code by adding boilerplate. On the course website you can download fully working implementations of some of the problems.

Recall that we occasionally annotate classes with **invariants** using the pseudo-code keyword **invariant**; **invariant** is **not** a valid Java keyword – that is why we highlight it in a different color – but we will use it to help make more explicit the behavior of classes. We also use `at(i)` or `at(i,j)` to indicate the number of threads that are at a location or range of locations. That's not Java either.

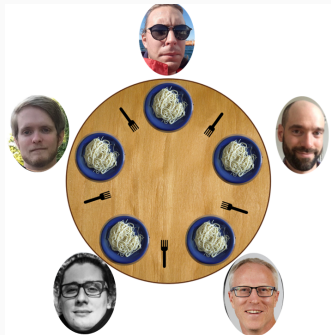
# Dining philosophers

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# The dining philosophers

The **dining philosophers** is a classic synchronization problem introduced by Dijkstra. It illustrates the problem of deadlocks using a colorful metaphor (by Hoare).

- Five philosophers are sitting around a dinner table, with a fork in between each pair of adjacent philosophers.
- Each philosopher alternates between thinking (**non-critical section**) and eating (**critical section**).
- In order **to eat**, a philosopher needs to pick up the **two forks** that lie to the philosopher's left and right.
- Since the forks are **shared**, there is a **synchronization** problem between philosophers (**threads**).



# Dining philosophers: the problem

```
interface Table {  
    // philosopher k picks up forks  
    void getForks(int k);  
    // philosopher k releases forks  
    void putForks(int k);  
}
```

**Dining philosophers** problem: implement `Table` such that:

- forks are held exclusively by one philosopher at a time,
- each philosopher only accesses adjacent forks.

**Properties** that a good solution should have:

- support an arbitrary number of philosophers,
- deadlock freedom,
- starvation freedom,
- reasonable efficiency: eating in parallel still possible.

# The philosophers

Each philosopher continuously alternate between thinking and eating; the table must guarantee proper synchronization when eating.

```
Table table; // table shared by all philosophers
```

---

```
    philosopherk
```

```
while (true) {  
    think();           // think  
    table.getForks(k); // wait for forks  
    eat();             // eat  
    table.putForks(k); // release forks  
}
```

# Left and right

For convenience, we introduce a consistent numbering scheme for forks and philosophers, in a way that it is easy to refer to the left or right fork of each philosopher.

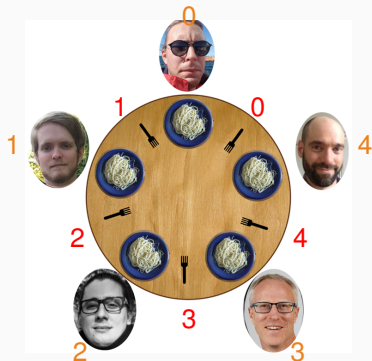
```
// in classes implementing Table:
```

```
// fork to the left of philosopher k
```

```
public int left(int k) {  
    return k;  
}
```

```
// fork to the right of philosopher k
```

```
public int right(int k) {  
    // N is the number of philosophers  
    return (k + 1) % N;  
}
```





# Dining philosophers with locks and semaphores

We use **semaphores** to implement mutual exclusion when philosophers access the forks. In fact, we only need **locks**:

```
Lock[] forks = new Lock[N]; // array of locks
```

- one lock per fork;
- `forks[i].lock()` to pick up fork `i`:  
 `forks[i]` is held if fork `i` is held;
- `forks[i].unlock()` to put down fork `i`:  
 `forks[i]` is available if fork `i` is available.

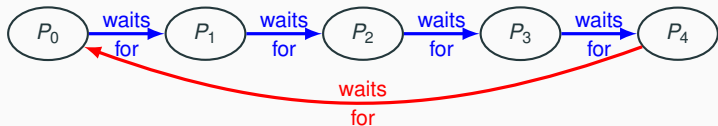
# Dining philosophers with semaphores: first attempt

In the first attempt, every philosopher picks up the **left** fork and then the **right** fork:

```
public class DeadTable implements Table {  
    Lock[] forks = new Lock[N]; ← all forks initially available  
  
    public void getForks(int k) {  
        // pick up left fork  
        forks[left(k)].lock();  
        // pick up right fork  
        forks[right(k)].lock();  
    }  
  
    public void putForks(int k) {  
        // put down left fork  
        forks[left(k)].unlock();  
        // put down right fork  
        forks[right(k)].unlock();  
    }  
}
```

# Dining philosophers with semaphores: first attempt

A **deadlock** may occur because of **circular waiting**:



```
public class DeadTable implements Table {
```

```
    Lock[] forks = new Lock[N];
```

```
    public void getForks(int k) {
```

```
        // pick up left fork
```

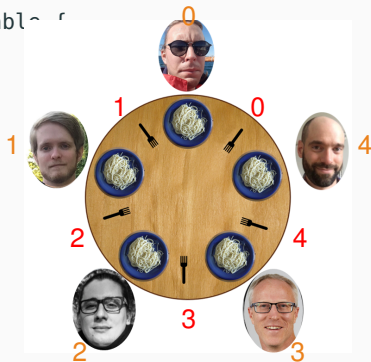
```
        forks[left(k)].lock();
```

```
        // pick up right fork
```

```
        forks[right(k)].lock();
```

```
    }
```

if all philosophers hold  
left fork: deadlock!



# Dining philosophers solution 1: breaking the symmetry

Having one philosopher pick up forks in a **different order** than the others is sufficient to break the symmetry, and thus to avoid deadlock.

```
public class AsymmetricTable implements Table {  
    Lock[] forks = new Lock[N];  
  
    public void getForks(int k) {  
        if (k == N) { // right before left  
            forks[right(k)].lock();  
            forks[left(k)].lock();  
        } else { // left before right  
            forks[left(k)].lock();  
            forks[right(k)].lock();  
        }  
    }  
}  
  
// putForks as in DeadTable
```



# Breaking symmetry to avoid deadlock

Breaking the symmetry is a **general strategy to avoid deadlock** when acquiring multiple shared resources:

- assign a **total order** between the shared resources

$$R_0 < R_1 < \dots < R_M$$

- a thread can try to obtain resource  $R_i$ , with  $i > j$ , only **after** it has successfully obtained resource  $R_j$

Recall the Coffman conditions from Lecture 2...

# Breaking symmetry to avoid deadlock

Breaking the symmetry is a **general strategy to avoid deadlock** when acquiring multiple shared resources:

- assign a **total order** between the shared resources  
 $R_0 < R_1 < \dots < R_M$
- a thread can try to obtain resource  $R_i$ , with  $i > j$ , only **after** it has successfully obtained resource  $R_j$

Recall the Coffman conditions from Lecture 2:

1. **mutual exclusion**: exclusive access to the shared resources,
2. **hold and wait**: request one resource while holding another,
3. **no preemption**: resources cannot forcibly be released,
4. **circular wait**: threads form a circular chain, each waiting for a resource that the next is holding.

Circular wait is a necessary condition for a deadlock to occur.

## Dining philosophers solution 2: bounding resources

**Limiting** the number of philosophers **active** at the table to  $M < N$  ensures that there are enough resources for everyone at the table, thus avoiding deadlock.

```
public class SeatingTable implements Table {
    Lock[] forks = new Lock[N];
    Semaphore seats = new Semaphore(M); // # available seats

    public void getForks(int k) {
        // get a seat
        seats.down();
        // pick up left fork
        forks[left(k)].lock();
        // pick up right fork
        forks[right(k)].lock();
    }

    public void putForks(int k) {
        // put down left fork
        forks[left(k)].unlock();
        // put down right fork
        forks[right(k)].unlock();
        // leave seat
        seats.up();
    }
}
```

# Starvation-free philosophers

The two solutions to the dining philosophers problem also guarantee **freedom from starvation**, under the assumption that locks/semaphores (and scheduling) are fair.

In the **asymmetric solution** (AsymmetricTable):

- if a philosopher  $P$  waits for a fork  $k$ ,  $P$  gets the fork as soon as  $P$ 's neighbor holding fork  $k$  releases it,
- $P$ 's neighbor eventually releases fork  $k$  because there are no deadlocks.

In the **bounded-resource solution** (SeatingTable):

- at most  $M$  philosophers are active at the table,
- the other  $N - M$  philosophers are waiting on `seats.down()`,
- the first of the  $M$  philosophers that finishes eating releases a seat,
- the philosopher  $P$  that has been waiting on `seats.down` proceeds,
- similarly to the asymmetric solution,  $P$  also eventually gets the forks.



# Producer-consumer

---

# Producer-consumer: overview

Producers and consumer exchange items through a **shared buffer**:

- **producers** asynchronously produce items and store them in the buffer,
- **consumers** asynchronously consume items after removing them from the buffer.



# Producer-consumer: the problem

```
interface Buffer<T> {  
    // add item to buffer; block if full  
    void put(T item);  
  
    // remove item from buffer; block if empty  
    T get();  
  
    // number of items in buffer  
    int count();  
}
```

**Producer-consumer** problem: implement Buffer such that:

- producers and consumers access the buffer in mutual exclusion,
- consumers block when the buffer is empty,
- producers block when the buffer is full (bounded buffer variant).

# Producer-consumer: desired properties

**Producer-consumer** problem: implement Buffer such that:

- producers and consumer access the buffer in mutual exclusion,
- consumers block when the buffer is empty,
- producers block when the buffer is full (bounded buffer variant).

Other properties that a good solution should have:

- support an arbitrary number of producers and consumers,
- deadlock freedom,
- starvation freedom.

# Producers and consumers

Producers and consumers continuously and asynchronously access the buffer, which must guarantee proper synchronization.

```
Buffer<Item> buffer;
```

---

producer<sub>*n*</sub>

```
while (true) {  
    // create a new item  
    Item item = produce();  
    buffer.put(item);  
}
```

consumer<sub>*m*</sub>

```
while (true) {  
    Item item = buffer.get();  
    // do something with 'item'  
    consume(item);  
}
```

# Unbounded shared buffer

```
public class UnboundedBuffer<T> implements Buffer<T> {
    Lock lock = new Lock(); // for exclusive access to buffer
    Semaphore nItems = new Semaphore(0); // number of items in buffer
    Collection storage = ...; // any collection (list, set, ...)
    invariant { storage.count() == nItems.count() + at(5,15-17); }

1  public void put(T item) {                public T get() {                12
2      lock.lock(); // lock                // wait until nItems > 0  13
3      // store item                        nItems.down();                14
4      storage.add(item);                  lock.lock(); // lock          15
5      nItems.up(); // update nItems        // retrieve item            16
6      lock.unlock(); // release            T item = storage.remove();    17
7  }                                        lock.unlock(); // release    18
8                                          return item;                  19
9  public int count() {                    }                               20
10     return nItems.count(); // locking here?
11 }
```

## Buffer: method put

```
public void put(T item) {  
    lock.lock(); // lock  
    // store item  
    storage.add(item);  
    // update nItems  
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## Buffer: method put

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    lock.unlock(); // release  
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```

signal to consumers waiting in get  
that they can proceed

Can we execute up after unlock?





## Buffer: method put

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public void put(T item) {  
    lock.lock(); // lock  
    // store item  
    storage.add(item);  
    // update nItems  
    nItems.up();  
    lock.unlock(); // release  
}
```

signal to consumers waiting in get  
that they can proceed

Can we execute up after unlock?

Executing up after unlock:

- No effects on other threads executing put: they only wait for lock.
- If a thread is waiting for `nItems > 0` in get: it does not have to wait again for lock just after it has been signaled to continue.
- If a thread is waiting for the lock in get: it may return with the buffer in a (temporarily) inconsistent state (broken invariant, but benign because temporary).

# Executing up after unlock

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6 }
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7 public T get() {  
8     nItems.down();  
9     lock.lock();  
10    T item = storage.remove();  
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12    return item;  
13 }
```

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+1	pc <sub>t</sub> : 3	pc <sub>u</sub> : 8	nItems: 1 buffer: ⟨x⟩
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+8	pc <sub>t</sub> : 5	done	nItems: 0 buffer: ⟨y⟩
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+3	pc <sub>t</sub> : 4	pc <sub>u</sub> : 9	nItems: 0 buffer: ⟨x, y⟩
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+5	pc <sub>t</sub> : 5	pc <sub>u</sub> : 10	nItems: 0 buffer: ⟨x, y⟩
+6	pc <sub>t</sub> : 5	pc <sub>u</sub> : 11	nItems: 0 buffer: ⟨y⟩
+7	pc <sub>t</sub> : 5	pc <sub>u</sub> : 12	nItems: 0 buffer: ⟨y⟩
+8	pc <sub>t</sub> : 5	done	nItems: 0 buffer: ⟨y⟩
+9	done	done	nItems: 1 buffer: ⟨y⟩

# Executing up after unlock

```
1 public void put(T item) {  
2     lock.lock();  
3     storage.add(item);  
4     lock.unlock();  
5     nItems.up();  
6 }
```

```
7 public T get() {  
8     nItems.down();  
9     lock.lock();  
10    T item = storage.remove();  
11    lock.unlock();  
12    return item;  
13 }
```

#	producer put	consumer get	SHARED
+1	pc <sub>t</sub> : 3	pc <sub>u</sub> : 8	nItems: 1 buffer: ⟨x⟩
+2	pc <sub>t</sub> : 3	pc <sub>u</sub> : 9	nItems: 0 buffer: ⟨x⟩
+3	pc <sub>t</sub> : 4	pc <sub>u</sub> : 9	nItems: 0 buffer: ⟨x, y⟩
+4	pc <sub>t</sub> : 5	pc <sub>u</sub> : 9	nItems: 0 buffer: ⟨x, y⟩
+5	pc <sub>t</sub> : 5	pc <sub>u</sub> : 10	nItems: 0 buffer: ⟨x, y⟩
+6	pc <sub>t</sub> : 5	pc <sub>u</sub> : 11	nItems: 0 buffer: ⟨y⟩
+7	pc <sub>t</sub> : 5	pc <sub>u</sub> : 12	nItems: 0 buffer: ⟨y⟩
+8	pc <sub>t</sub> : 5	done	nItems: 0 buffer: ⟨y⟩
+9	done	done	nItems: 1 buffer: ⟨y⟩

## Buffer: method get

```
public T get() {  
    nItems.down();           // wait until nItems > 0  
    lock.lock();           // lock  
    T item = storage.remove(); // retrieve item  
    lock.unlock();         // release  
    return item;  
}
```

What happens if another thread gets the lock just after the current threads has decremented the semaphore `nItems`?

- If the other thread is a producer, it does not matter: as soon as `get` resumes execution, there will be one element in storage to remove.
- If the other thread is a consumer, it must have synchronized with the current thread on `nItems.down()`, and the order of removal of elements from the buffer does not matter.

## Buffer: method get

```
public T get() {  
    nItems.down();  
    lock.lock();  
    T item = storage.remove();  
    lock.unlock();  
    return item;  
}
```

Can we execute down after lock?

## Buffer: method get

```
public T get() {  
    nItems.down();  
    lock.lock();  
    T item = storage.remove();  
    lock.unlock();  
    return item;  
}
```

Can we execute down after lock?

Executing down after lock:

- if the buffer is empty when locking, there is a **deadlock!**

# Bounded shared buffer

```
public class BoundedBuffer<T> implements Buffer<T> {  
    Lock lock = new Lock(); // for exclusive access to buffer  
    Semaphore nItems = new Semaphore(0); // # items in buffer  
    Semaphore nFree = new Semaphore(N); // # free slots in buffer  
    Collection storage = ...; // any collection (list, set, ...)  
    invariant { storage.count() == nItems.count() +  
                + at(6,13-15) == N - nFree.count() - at(4-6,16) ; }
```

```
1 public void put(T item) {  
2     // wait until nFree > 0  
3     nFree.down();  
4     lock.lock(); // lock  
5     // store item  
6     storage.add(item);  
7     nItems.up(); // update nItems  
8     lock.unlock(); // release  
9 }  
  
    public T get() {  
        // wait until nItems > 0  
        nItems.down();  
        lock.lock(); // lock  
        // retrieve item  
        T item = storage.remove();  
        nFree.up(); // update nFree  
        lock.unlock(); // release  
        return item;  
    }
```

# Bounded shared buffer


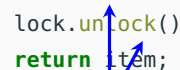
```
public class BoundedBuffer<T> implements Buffer<T> { size of buffer
    Lock lock = new Lock(); // for exclusive access to buffer
    Semaphore nItems = new Semaphore(0); // # items in buffer
    Semaphore nFree = new Semaphore(N); // # free slots in buffer
    Collection storage = ...; // any collection (list, set, ...)
    invariant { storage.count() == nItems.count() +
                + at(6,13-15) == N ← nFree.count() - at(4-6,16) ; }

1 public void put(T item) { public T get() { 10
2     // wait until nFree > 0 // wait until nItems > 0 11
3     nFree.down(); nItems.down(); 12
4     lock.lock(); // lock lock.lock(); // lock 13
5     // store item // retrieve item 14
6     storage.add(item); T item = storage.remove(); 15
7     nItems.up(); // update nItems nFree.up(); // update nFree 16
8     lock.unlock(); // release lock.unlock(); // release 17
9 } return item; 18
} 19
```



# Bounded shared buffer

```
public class BoundedBuffer<T> implements Buffer<T> {  
    Lock lock = new Lock(); // for exclusive access to buffer  
    Semaphore nItems = new Semaphore(0); // # items in buffer  
    Semaphore nFree = new Semaphore(N); // # free slots in buffer  
    Collection storage = ...; // any collection (list, set, ...)  
    invariant { storage.count() == nItems.count() +  
                + at(6,13-15) == N - nFree.count() - at(4-6,16) ; }
```

```
1 public void put(T item) {                               public T get() {                               10  
2     // wait until nFree > 0                               // wait until nItems > 0                               11  
3     nFree.down(); may deadlock                               nItems.down(); may deadlock                               12  
4     lock.lock(); if swapped                               lock.lock(); if swapped                               13  
5     // store item                                           // retrieve item                                           14  
6     storage.add(item);                                       T item = storage.remove();                               15  
7     nItems.up(); // update nItems                             nFree.up(); // update nFree                               16  
8     lock.unlock(); // release                               lock.unlock(); // release                               17  
9 } OK to swap                                return item;                               18  
} OK to swap                                19
```

# Waiting on multiple conditions?

The operations offered by semaphores do **not support** waiting on **multiple conditions** (not empty and not full in our case) using one semaphore.

Busy-waiting on the semaphore will **not work**:

```
// wait until there is space in the buffer  
while (!(nItems.count() < N)) {};  
// the buffer may be full again when locking!  
lock.lock(); // lock  
// store item  
storage.add(item);  
nItems.up(); // update nItems  
lock.unlock(); // release
```

# Barriers

---

## Barriers (also called rendezvous)

A **barrier** is a form of synchronization where there is a point (the barrier) in a program's execution that all threads in a group have to reach before any of them is allowed to continue



## Barriers (also called rendezvous)

A **barrier** is a form of synchronization where there is a point (the barrier) in a program's execution that all threads in a group have to reach before any of them is allowed to continue



A **solution** to the barrier synchronization problem **for 2 threads** using binary semaphores.

```
Semaphore[] done = {new Semaphore(0), new Semaphore(0)};
```

---

$t_0$

```
// code before barrier  
done[ $t_0$ ].up(); // t done  
done[ $t_1$ ].down(); // wait u  
// code after barrier
```

$t_1$

```
// code before barrier  
done[ $t_1$ ].up(); // u done  
done[ $t_0$ ].down(); // wait t  
// code after barrier
```

# Barriers (also called rendezvous)

A **barrier** is a form of synchronization where there is a point (the barrier) in a program's execution that all threads in a group have to reach before any of them is allowed to continue



A **solution** to the barrier synchronization problem for 2 threads using binary semaphores. **capacity 0 forces up before first down**

```
Semaphore[] done = {new Semaphore(0), new Semaphore(0)};
```

*t<sub>0</sub>*

```
// code before barrier  
done[t0].up(); // t done  
done[t1].down(); // wait u  
// code after barrier
```

up done unconditionally

*t<sub>1</sub>*

```
// code before barrier  
done[t1].up(); // u done  
done[t0].down(); // wait t  
// code after barrier
```

down waits until the other thread has reached the barrier

## Barriers: variant

The solution still works if  $t_0$  performs down before up – or, symmetrically, if  $t_1$  does the same.

```
Semaphore[] done = new Semaphore(0), new Semaphore(0);
```

---

$t_0$

```
// code before barrier  
done[t1].down(); // wait u  
done[t0].up();   // t done  
// code after barrier
```

$t_1$

```
// code before barrier  
done[t1].up();   // u done  
done[t0].down(); // wait t  
// code after barrier
```

This solution is, however, a bit less efficient: the last thread to reach the barrier has to stop and yield to the other (one more context switch).

# Barriers: deadlock!

The solution **deadlocks** if **both**  $t_0$  and  $t_1$  perform down before up.

```
Semaphore[] done = new Semaphore(0), new Semaphore(0);
```

---

$t_0$	$t_1$
<pre>// code before barrier</pre>	<pre>// code before barrier</pre>
<pre>done[t<sub>1</sub>].down(); // wait u</pre>	<pre>done[t<sub>0</sub>].down(); // wait t</pre>
<pre>done[t<sub>0</sub>].up(); // t done</pre>	<pre>done[t<sub>1</sub>].up(); // u done</pre>
<pre>// code after barrier</pre>	<pre>// code after barrier</pre>

There is a **circular waiting**, because no thread has a chance to signal to the other that it has reached the barrier.



## Barriers with $n$ threads

Keeping track of  $n$  threads reaching the barrier:

- `nDone`: number of threads that have reached the barrier
- `lock`: to update `nDone` atomically
- `open`: to release the waiting threads (“opening the barrier”)

```
int nDone = 0; // number of done threads
Lock lock = new Lock(); // mutual exclusion for nDone
Semaphore open = new Semaphore(0); // 1 iff barrier is open
```

---

```
thread  $t_k$  total number of expected threads
// code before barrier
lock.lock(); // lock nDone
nDone = nDone + 1; // I'm done
if (nDone == n) open.up(); // I'm the last: we can go!
lock.unlock(); // unlock nDone
open.down(); // proceed when possible
open.up(); // let the next one go
// code after barrier
```

## Barriers with $n$ threads: variant

```
int nDone = 0; // number of done threads
Lock lock = new Lock(); // mutual exclusion for nDone
Semaphore open = new Semaphore(0); // 1 iff barrier is open
```

---

```
thread  $t_k$ 
// code before barrier
lock.lock(); // lock nDone
nDone = nDone + 1; // I'm done
lock.unlock(); // unlock nDone
if (nDone == n) open.up(1); // I'm the last: we can go!
open.down(); // proceed when possible
open.up(); // let the next one go
// code after barrier
```

can we open the barrier after unlock?



## Barriers with $n$ threads: variant

```
int nDone = 0; // number of done threads
Lock lock = new Lock(); // mutual exclusion for nDone
Semaphore open = new Semaphore(0); // 1 iff barrier is open
```

---

```
thread  $t_k$ 
// code before barrier
lock.lock(); // lock nDone
nDone = nDone + 1; // I'm done
lock.unlock(); // unlock nDone
if (nDone == n) open.up(1); // I'm the last: we can go!
open.down(); // proceed when possible
open.up(); // let the next one go
// code after barrier
```

can we open the barrier after unlock?

- in general, reading a shared variable outside a lock may give an inconsistent value
- in this case, however, only after the last thread has arrived can any thread read `nDone == n`, because `nDone` is only incremented


## Barriers with $n$ threads: variant

Signaling after unlocking follows the rule of thumb of minimizing the operations under lock (provided it does not affect correctness!).

```
int nDone = 0; // number of done threads
Lock lock = new Lock(); // mutual exclusion for nDone
Semaphore open = new Semaphore(0); // 1 iff barrier is open
```

---

```
        thread  $t_k$ 
// code before barrier
lock.lock();           // lock nDone
nDone = nDone + 1;    // I'm done
lock.unlock();        // unlock nDone
if (nDone == n) open.up(); // I'm the last: we can go!
open.down();          // proceed when possible
open.up();            // let the next one go
// code after barrier
```



such pairs of wait/signal are called **turnstiles**

# Reusable barriers

```
interface Barrier {  
    // block until expect() threads have reached barrier  
    void wait();  
  
    // number of threads expected at the barrier  
    int expect();  
}
```

**Reusable barrier:** implement Barrier such that:

- a thread blocks on `wait` until all threads have reached the barrier
- after `expect()` threads have executed `wait`, the barrier is closed again

# Threads at a reusable barrier

Threads **continuously approach** the barrier, which must guarantee that they synchronize each access.

```
Barrier barrier = new Barrier(n); // barrier for n threads
```

---

```
        threadk  
while (true) {  
    // code before barrier  
    barrier.wait(); // synchronize at barrier  
    // code after barrier  
}
```

## Reusable barriers: first attempt

```
public class NonBarrier1 implements Barrier {
    int nDone = 0; // number of done threads
    Semaphore open = new Semaphore(0);
    final int n;

    // initialize barrier for 'n' threads
    NonBarrier1(int n) {
        this.n = n;
    }

    // number of threads expected at the barrier
    int expect() {
        return n;
    }
    // continues in the next slide
}
```

## Reusable barriers: first attempt (cont'd)

```
public class NonBarrier implements Barrier {
    int nDone = 0; // number of done threads
    Semaphore open = new Semaphore(0);
    final int n;

    public void wait() {
        synchronized(this) { nDone += 1; } // I'm done
        if (nDone == n) open.up(); // I'm the last arrived:
                                   // we can go!
        open.down() // proceed when possible
        open.up() // let the next one go
        synchronized(this) { nDone -= 1; } // I've gone through
        if (nDone == 0) open.down(); // I'm the last through:
    } // close barrier!
```



## Reusable barriers: first attempt (cont'd)

```
public class NonBarrier implements Barrier {  
    int nDone = 0; // number of done threads  
    Semaphore open = new Semaphore(0);  
    final int n;
```

What if  $n$  threads block here until `nDone == n`?

```
public void wait() {  
    synchronized(this) { nDone += 1; } // I'm done  
    if (nDone == n) open.up(); // I'm the last arrived:  
                                // we can go!  
    open.down() // proceed when possible  
    open.up() // let the next one go  
    synchronized(this) { nDone -= 1; } // I've gone through  
    if (nDone == 0) open.down(); // I'm the last through:  
    } // close barrier!
```

## Reusable barriers: first attempt (cont'd)

```
public class NonBarrier implements Barrier {  
    int nDone = 0; // number of done threads  
    Semaphore open = new Semaphore(0);  
    final int n;
```

What if  $n$  threads block here until `nDone == n`?

```
public void wait() {  
    synchronized(this) { nDone += 1; } // I'm done  
    if (nDone == n) open.up(); // I'm the last arrived:  
                                // we can go!  
    open.down() // proceed when possible  
    open.up() // let the next one go  
    synchronized(this) { nDone -= 1; } // I've gone through  
    if (nDone == 0) open.down(); // I'm the last through:  
    } // close barrier!
```

More than one thread may open the barrier (the first `open.up()`): this was not a problem in the non-reusable version, but now some threads may be executing `wait` again before the barrier is closed again!

## Reusable barriers: first attempt (cont'd)

```
public class NonBarrier implements Barrier {  
    int nDone = 0; // number of done threads  
    Semaphore open = new Semaphore(0);  
    final int n;
```

What if  $n$  threads block here until `nDone == 0`?

```
public void wait() {  
    synchronized(this) { nDone += 1; } // I'm done  
    if (nDone == n) open.up(); // I'm the last arrived:  
                                // we can go!  
    open.down(); // proceed when possible  
    open.up(); // let the next one go  
    synchronized(this) { nDone -= 1; } // I've gone through  
    if (nDone == 0) open.down(); // I'm the last through:  
                                // close barrier!  
}
```

## Reusable barriers: first attempt (cont'd)

```
public class NonBarrier implements Barrier {  
    int nDone = 0; // number of done threads  
    Semaphore open = new Semaphore(0);  
    final int n;
```

What if  $n$  threads block here until  $nDone == 0$ ?

```
public void wait() {  
    synchronized(this) { nDone += 1; } // I'm done  
    if (nDone == n) open.up(); // I'm the last arrived:  
                                // we can go!  
    open.down(); // proceed when possible  
    open.up(); // let the next one go  
    synchronized(this) { nDone -= 1; } // I've gone through  
    if (nDone == 0) open.down(); // I'm the last through:  
    } // close barrier!
```

More than one thread may try to close the barrier (the last `open.down()`): **deadlock!**

## Reusable barriers: second attempt

```
public class NonBarrier2 implements Barrier {
    // same variables as in NonBarrier1

    public void wait() {
        synchronized(this)
        { nDone += 1;                               // I'm done
          if (nDone == n) open.up(); }             // open barrier
        open.down()                                // proceed when possible
        open.up()                                  // let the next one go
        synchronized(this)
        { nDone -= 1;                               // I've gone through
          if (nDone == 0) open.down(); }           // close barrier
    }
}
```

## Reusable barriers: second attempt

```
public class NonBarrier2 implements Barrier {
    // same variables as in NonBarrier1

    public void wait() {
        synchronized(this)
        { nDone += 1;                // I'm done
          if (nDone == n) open.up(); } // open barrier
        open.down()                // proceed when possible
        open.up()                  // let the next one go
        synchronized(this)
        { nDone -= 1;                // I've gone through
          if (nDone == 0) open.down(); } // close barrier
    }
}
```

Now multiple signaling is not possible. But a fast thread may **race through** the whole method, and re-enter it before the barrier has been closed, thus **getting ahead** of the slower threads – which are still in the previous iteration of the barrier.

## Reusable barriers: second attempt (cont'd)

A fast thread may **race through** the whole method, and re-enter it before the barrier has been closed, thus **getting ahead** of the slower threads – which are still in the previous iteration of the barrier. This is not prevented by strong semaphores: it occurs because the last thread through leaves the gate open (calls `open.up()`)

```
1 public class NonBarrier2 {
2   public void wait() {
3     synchronized(this)
4     {nDone += 1;
5     if (nDone == n) open.up();}
6     open.down()
7     open.up()
8     synchronized(this)
9     {nDone -= 1;
10    if (nDone == 0) open.down();}
11 }
```

- (a) All  $n$  threads are at 8, with `open.count() == 1`
- (b) The fastest thread  $t_f$  completes `wait` and re-enters it with `nDone == n - 1`
- (c) Thread  $t_f$  reaches 6 with `nDone == n`, which it can execute because `open.count() > 0`
- (d) Thread  $t_f$  reaches 8 again, but it is one iteration ahead of all other threads!

# Reusable barriers: correct solution



Photo by Photnart: Heidelberg Lock, Germany.



## Reusable barriers: correct solution

```
public class SemaphoreBarrier implements Barrier {
    int nDone = 0; // number of done threads
    Semaphore gate1 = new Semaphore(0); // first gate
    Semaphore gate2 = new Semaphore(1); // second gate
    final int n;

    // initialize barrier for 'n' threads
    SemaphoreBarrier(int n) {
        this.n = n;
    }

    // number of threads expected at the barrier
    int expect() {
        return n;
    }
    // continues in the next slide
}
```

## Reusable barriers: correct solution

```
public class SemaphoreBarrier implements Barrier {  
    int nDone = 0; // number of done threads  
    Semaphore gate1 = new Semaphore(0); // first gate  
    Semaphore gate2 = new Semaphore(1); // second gate
```

```
void approach() {  
    synchronized (this) {  
        nDone += 1; // arrived  
        if (nDone == n) // if last in:  
        { gate1.up(); // open gate1  
          gate2.down(); } // close gate2  
    }  
    gate1.down(); // pass gate1  
    gate1.up(); // let next pass  
}  
  
void leave() {  
    synchronized (this) {  
        nDone -= 1; // going out  
        if (nDone == 0) // if last out:  
        { gate2.up(); // open gate2  
          gate1.down(); } // close gate1  
    }  
    gate2.down(); // pass gate2  
    gate2.up(); // let next pass  
}
```

```
public void wait() { approach(); leave(); }
```

## Reusable barriers: improved solution

If the semaphores support **adding  $n$  to the counter** at once, we can write a barrier with fewer semaphore accesses.

both gates initially closed

```
public class NSemaphoreBarrier extends SemaphoreBarrier {  
    Semaphore gate1 = new Semaphore(0); // first gate  
    Semaphore gate2 = new Semaphore(0); // second gate
```

```
void approach() {  
    synchronized (this) {  
        nDone += 1;  
        if (nDone == n)  
            gate1.up(n); // open gate1  
                           for n threads  
    }  
    gate1.down(); // pass gate1  
    // last thread here closes gate1  
}
```

```
void leave() {  
    synchronized (this) {  
        nDone -= 1;  
        if (nDone == 0)  
            gate2.up(n); // open gate2  
                           for n threads  
    }  
    gate2.down();  
    // last thread here closes gate2  
}
```

Java semaphores support adding  $n$  to counter (`release(n)`). Anyway, `up(n)` need not be atomic, so we can also implement it with a loop.

# Readers-writers

---

# Readers-writers: overview

Readers and writers concurrently access **shared data**:

- **readers** may execute concurrently with other readers, but need to exclude writers
- **writers** need to exclude both readers and other writers

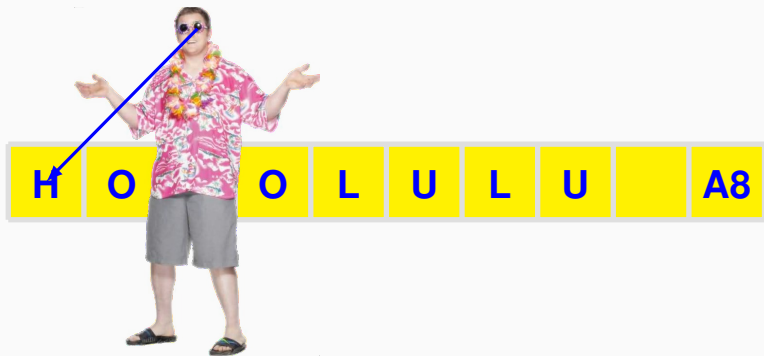
The problem captures situations common in databases, filesystems, and other situations where accesses to shared data may be **inconsistent**.



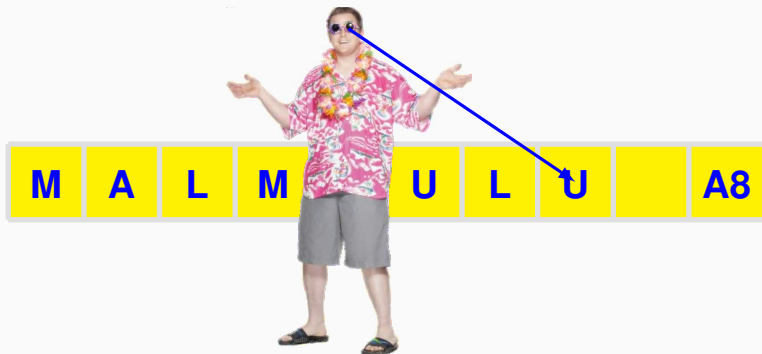
What's the gate for the flight to Honolulu?

H	O	N	O	L	U	L	U		A8
---	---	---	---	---	---	---	---	--	----

# What's the gate for the flight to Honolulu?



# What's the gate for the flight to Honolulu?





# What's the gate for the flight to Honolulu?

M A L M Ö [ ] [ ] [ ] Z6



# Readers-writers: the problem

```
interface Board<T> {  
    // write message 'msg' to board  
    void write(T msg);  
    // read current message on board  
    T read();  
}
```

**Readers-writers** problem: implement Board data structure such that:

- multiple reader can operate concurrently
- each writer has exclusive access

Invariant:  $\#WRITERS = 0 \vee (\#WRITERS = 1 \wedge \#READERS = 0)$

Other properties that a good solution should have:

- support an arbitrary number of readers and writers
- no starvation of readers or writers

# Readers and writers

Readers and writers continuously and asynchronously try to access the board, which must guarantee proper synchronization.

```
Board<Message> board;
```

---

*reader<sub>n</sub>*

```
while (true) {  
    // read message from board  
    Message msg = board.read();  
    // do something with 'msg'  
    process(msg);  
}
```

*writer<sub>m</sub>*

```
while (true) {  
    // create a new message  
    Message msg = create();  
    // write 'msg' to board  
    board.write(msg);  
}
```

# Readers-writers board: write

```
public class SyncBoard<T> implements Board<T> {
    int nReaders = 0; // # readers on board
    Lock lock = new Lock(); // for exclusive access to nReaders
    Semaphore empty = new Semaphore(1); // 1 iff no active threads
    T message; // current message

    public void write(T msg) {
        // get exclusive access
        empty.down();
        message = msg; // write (cs)
        // release board
        empty.up();
    }
}
```

```
invariant { nReaders == 0
            ⇔ empty.count() == 1 }
```

## Readers-writers board: read

```
public class SyncBoard<T> implements Board<T> {
    int nReaders = 0; // # readers on board
    Lock lock = new Lock(); // for exclusive access to nReaders
    Semaphore empty = new Semaphore(1); // 1 iff no active threads
    T message; // current message

    public T read() {
        lock.lock(); // lock to update nReaders
        if (nReaders == 0) empty.down(); // if first reader, set not empty
        nReaders += 1; // update active readers
        lock.unlock(); // release lock to nReaders
        T msg = message; // read (critical section)
        lock.lock(); // lock to update nReaders
        nReaders -= 1; // update active readers
        if (nReaders == 0) empty.up(); // if last reader, set empty
        lock.unlock(); // release lock to nReaders
        return msg;
    }
}
```

# Properties of the readers-writers solution

We can check the following **properties** of the solution:

- empty is a binary semaphore,
- when a writer is running, no reader can run,
- one reader waiting for a writer to finish also locks out other readers,
- a reader signals “empty” only when it is the last reader to leave the board,
- deadlock is not possible (no circular waiting).

However, **writers can starve**: as long as readers come and go with at least one reader always active, writers are shut out of the board.

# Readers-writers board without starvation

```
public class FairBoard<T> extends SyncBoard<T> {
    // held by the next thread to go
    Semaphore baton = new Semaphore(1, true); // fair binary sem.

    public T read() {
        // wait for my turn
        baton.down();
        // release a waiting thread
        baton.up();
        // read() as in SyncBoard
        return super.read();
    }

    public void write(T msg) {
        // wait for my turn
        baton.down();
        // write() as in SyncBoard
        super.write(msg);
        // release a waiting thread
        baton.up();
    }
}
```

# Readers-writers board without starvation

```
public class FairBoard<T> extends SyncBoard<T> {
    // held by the next thread to go
    Semaphore baton = new Semaphore(1, true); // fair binary sem.

    public T read() {
        // wait for my turn
        baton.down();
        // release a waiting thread
        baton.up();
        // read() as in SyncBoard
        return super.read();
    }

    public void write(T msg) {
        // wait for my turn
        baton.down();
        // write() as in SyncBoard
        super.write(msg);
        // release a waiting thread
        baton.up();
    }
}
```

Now **writers do not starve**: suppose a writer is waiting that all active readers leave: it waits on `empty.down()` while holding the baton. If new readers arrive, they are shut out waiting for the baton. As soon as the active readers terminate and leave, the writer is signaled empty, and thus it gets exclusive access to the board.



# Readers-writers with priorities

The starvation free solution we have presented gives all threads the **same priority**: assuming a fair scheduler, writers and readers take turn as they try to access the board.

In some applications it might be preferable to enforce **difference priorities**:

- $R = W$ : readers and writers have the same priority (as in FairBoard)
- $R > W$ : readers have higher priority than writers (as in SyncBoard)
- $W > R$ : writers have higher priority than readers

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